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Josephson-coupled layered superconductors with two order parameters: I. Upper critical magnetic field

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Abstract. By using the Gor'kov–Nambu microscopic theory, an expression for the free-energy functional is obtained for Josephson-coupled layered superconductors with two order parameters corresponding to intra- and interlayer Cooper pairs. The existence of two order parameters in a system leads to the appearance of new terms in the free-energy functional known as the 'Josephson' terms and the Lifshitz invariant. Of these, the Josephson terms characterize the tunnelling of both intra- and interlayer Cooper pairs between neighbouring superconducting sheets.

The upper critical field, H_{c2}, is studied for the superconductors under investigation. It is found that a magnetic field parallel to the superconducting layers does not destroy the superconductivity for temperatures $T < T^*$, where T^* depends on the resonance integral between two neighbouring superconducting layers. This effect gives rise to a positive curvature in the temperature dependence of $H_{c2}^{\parallel}$.

1. Introduction

The anisotropy of the recently synthesized bismuth- and thallium-based superconductors is shown to increase with the number of CuO_2 sheets between Bi–O of Tl–O layers [1–3]. In these superconductors the coherence length $\mathcal{E}_{\perp_0}$, along the direction perpendicular to the CuO_2 layers is estimated to be comparable with the distance d between superconducting planes [4].

Moreover, experimental investigations of superconducting fluctuations in the copper oxide materials exhibit dimensional crossover from three-dimensional (3D) behaviour around T_c to two-dimensional (2D) behaviour away from T_c [5, 6]. All these facts confirm that the Josephson coupling between the layers is realized [7] in high-T_c superconductors with Bi and Tl.

Strongly anisotropic superconductors (SC) with Josephson coupling between the layers have been studied by many authors [8–12] mainly based on the Lawrence–Doniach (LD) free-energy functional [8].

Layered superconductors are described by a model that proposes the electronic motion to be free and isotropic inside the layers, while the tight-binding approximation is appropriate to characterize the interlayer motion. Then the electron spectrum will be given as

$$\mathcal{E}(p, p_z) = p^2/(2m) + t_\perp[1 - \cos(p_z d)] \tag{1}$$

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where $t_\perp$ is the resonance integral for an electron to move between the layers.

For the case of $t_\perp > kT_c^{(0)}$, where $T_c^{(0)}$ is the critical temperature evaluated by the mean-field theory, the anisotropic properties of superconductors are described by introducing the anisotropic effective mass in Ginzburg–Landau (GL) theory [13, 14]. In the opposite case of $t_\perp < kT_c^{(0)} < \mathcal{E}_F$, where $\mathcal{E}_F$ is the Fermi energy of a free electron inside a superconducting plane, an electron travels a distance of the order of the correlation length before tunnelling to the neighbouring layer. In this case the Josephson-like coupling between the SC layers is just realized. Then the differential–difference GL equations are appropriate to describe the layered SC [7–12].

In all the previous works devoted to the Josephson-coupled model of layered SC [7–12], an electron pair is assumed to be formed inside each SC layer. Cooper pairs may tunnel from one layer to another. A superconductor becomes effectively two-dimensional with the weakening of interlayer coupling. The order parameter phase fluctuations should destroy off-diagonal long-range order (ODLRO) in a 2D superconductor [15]. However, the existence of Berezinshii–Kosterlitz–Thousless topological defects sets a 'quasi-long-range' order in a system [16, 17]. An interlayer pairing mechanism (see [18–24] and references therein) is suggested to stabilize ODLRO [19]. It has been shown that an additional channel of pairing can enhance the superconducting critical temperature. The upper critical magnetic field H_{c2} has also been calculated [21, 22] in the case of $t_\perp > kT_c^{(0)}$.

This paper deals with the superconducting properties of layered systems with inter- and intralayer electron pairing in the case of $t_\perp < kT_c^{(0)} < \mathcal{E}_F$ where Josephson coupling between SC layers is present.

The outline of the paper is as follows. In section 2 we obtain the Ginzburg–Landau free-energy functional by using the Gor'kov–Nambu formalism [25, 26]. In section 3, the upper critical magnetic field H_{c2} is calculated in both directions, parallel and perpendicular to the SC layers. The formulae obtained for H_{c2} make it possible to understand the positive curvature in the temperature dependence of H_{c2}. In section 4 the discussion of the results is given.

2. Ginzburg–Landau free-energy functional for layered superconductors with intra- and interlayer pairing

Layered superconductors may be described as regularly arranged conducting planes. There exists a finite probability of electron tunnelling between neighbouring layers. Such a layered SC is characterized by the following Hamiltonian:

$$\hat{H} = \hat{H}_0 + \hat{H}_{t_\perp} + \hat{H}_{e-e} \tag{2}$$

with

$$\hat{H}_0 = d\sum_{j,\sigma}\int d^2r\left\{\psi_{j\sigma}^+(r)\left[-\frac{\hbar^2}{2m}\left(\frac{\partial}{\partial r} - i\frac{e}{\hbar c}A(r,j)\right)^2 - \mathcal{E}_F\right]\Psi_{j\sigma}(r)\right\} \tag{2a}$$

$$\hat{H}_{t_\perp} = d\frac{t_\perp}{2}\sum_{j,\sigma}\int d^2r\left[\psi_{j\sigma}^+(r)\Psi_{j+1,\sigma}(r)\exp\left(-i\frac{e}{\hbar c}\int_{jd}^{(j+1)d}A_z(r,\kappa)\,d\kappa\right) + \mathrm{HC}\right] \tag{2b}$$

$$\hat{H}_{e-e} = \frac{d}{2}\sum_{\substack{j,l,\\ \sigma,\sigma'}}\int d^2r\int d^2r'\Psi_{j\sigma}^+(r)\Psi_{l\sigma'}^+(r')V_{jl}(r-r')\Psi_{l\sigma'}(r')\Psi_{j\sigma}(r) \tag{2c}$$

Here, $\Psi^{+}_{j\sigma}(\boldsymbol{r})$ is the creation operator of the electron with spin σ at the point $\boldsymbol{r}$ of the jth layer; $t_{\perp}$ is the transfer integral between the nearest planes; and $\boldsymbol{A}$ and A_z are the magnetic vector potentials parallel and perpendicular to the layers, respectively. The potential $V_{jl}(\boldsymbol{r}-\boldsymbol{r}')$ in (2*c*) represents the interaction between two electrons placed at points $\boldsymbol{r}$ and $\boldsymbol{r}'$ of the layers j and l. Without specifying the origin of the electron–electron (e–e) attractive interaction we shall represent $V_{jl}(\boldsymbol{r}-\boldsymbol{r}')$ as:

$$V_{jl}(\boldsymbol{r}-\boldsymbol{r}') = [V_0\delta_{jl} + V_1(\delta_{j,l+1}+\delta_{j,l-1})]\delta(\boldsymbol{r}-\boldsymbol{r}') \tag{3}$$

and

$$V_i = -|V_i| \qquad i = 0, 1.$$

Such interaction is analogous to the phonon-mediated e–e attraction in the Bardeen–Cooper–Schrieffer (BCS) theory. Therefore here, as in the BCS theory, only the electrons in the vicinity of the Fermi surface are assumed to take part in the pairing process. The existence of the interlayer pairing, V_1, makes this model a non-point-interaction model. As a result, triplet pairing of two electrons on the nearest-neighbouring layers may become possible. However, here we shall consider only spin-singlet pairing. To obtain the Ginzburg–Landau free-energy functional by using microscopic theory we shall follow Gor'kov's method [25].

The normal $G^{\alpha\beta}_{jj'}(x,x')$ and anomalous $F^{\alpha\beta}_{jj'}(x,x')$ Green functions are defined as

$$G^{\alpha\beta}_{jj'}(x,x') = -\langle T_\tau(\Psi_{j\alpha}(x)\Psi^{+}_{j'\beta}(x'))\rangle \tag{4}$$

$$F^{\alpha\beta}_{jj'}(x,x') = \langle T_\tau(\Psi_{j\alpha}(x)\Psi_{j'\beta}(x'))\rangle \tag{5}$$

where $x=\{\boldsymbol{r},\tau\}$ denotes both coordinate $\boldsymbol{r}$ and imaginary 'time' τ. T_τ is the chronological operator.

By using the equation of motion for an operator $\Psi_{j\alpha}(x)$, given in the Heizenberg representation, we may obtain the equations for $G^{\alpha\beta}_{jj'}(x,x')$ and $F^{\alpha\beta}_{jj'}(x,x')$. In the absence of a spin-dependent interaction, the spin indices of the normal and anomalous Green functions can be omitted, by using the following relations:

$$G^{\alpha\beta}_{ij}(x,x') = \delta_{\alpha\beta}G_{ij}(x,x') \tag{6a}$$

$$F^{\alpha\beta}_{ij}(x,x') = \delta_{\alpha,-\beta}F_{ij}(x,x'). \tag{6b}$$

Then the equations for $G_{ij}(x,x')$ and $F_{ij}(x,x')$ are represented by the following formulae:

$$\begin{aligned}
&\left[-\frac{\partial}{\partial\tau}+\frac{\hbar^2}{2m}\left(\frac{\partial}{\partial\boldsymbol{r}}-\mathrm{i}\frac{e}{\hbar c}\boldsymbol{A}(\boldsymbol{r},j)\right)^2+\mathcal{E}_{\mathrm{F}}\right]G_{jj'}(x,x')\\
&\quad-\frac{t_\perp}{2}\left[\exp\left(-\mathrm{i}\frac{e}{\hbar c}\int_{jd}^{(j+1)d}A_z(\boldsymbol{r},\kappa)\,\mathrm{d}\kappa\right)G_{j+1,j'}(x,x')\right.\\
&\quad\left.+\exp\left(-\mathrm{i}\frac{e}{\hbar c}\int_{jd}^{(j-1)d}A_z(\boldsymbol{r},\kappa)\,\mathrm{d}\kappa\right)G_{j-1,j'}(x,x')\right]+V_0F_{jj}(0^+)F^{+}_{jj'}(x,x')\\
&\quad+V_1[F_{j,j+1}(0^+)F_{j+1,j'}(x,x')+F_{j,j-1}(0^+)F^{+}_{j-1,j'}(x,x')]=\delta_{jj'}\delta(\boldsymbol{r}-\boldsymbol{r}')
\end{aligned} \tag{7}$$

$$\left[\frac{\partial}{\partial\tau}-\frac{\hbar^2}{2m}\left(\frac{\partial}{\partial \boldsymbol{r}}+\mathrm{i}\frac{e}{\hbar c}\boldsymbol{A}(\boldsymbol{r},j)\right)^2-\mathcal{E}_{\mathrm{F}}\right]F^+_{j,j'}(x,x')$$
$$+\frac{t_\perp}{2}\left[\exp\left(-\mathrm{i}\frac{e}{\hbar c}\int_{(j+1)d}^{jd}A_z(\boldsymbol{r},\kappa)\,\mathrm{d}\kappa\right)F^+_{j+1,j'}(x,x')\right.$$
$$\left.+\exp\left(-\mathrm{i}\frac{e}{\hbar c}\int_{(j-1)d}^{jd}A_z(\boldsymbol{r},\kappa)\,\mathrm{d}\kappa\right)F^+_{j-1,j'}(x,x')\right]+V_0F^+_{jj}(0^+)G_{jj'}(x,x')$$
$$+V_1[F^+_{j,j+1}(0^+)G_{j+1,j'}(x,x')+F^+_{j,j-1}(0^+)G_{j-1,j'}(x,x')]=0. \qquad (8)$$

The equation (7) for a free particle (i.e. V_0, $V_1=0$) without an external magnetic field is solved easily in a momentum representation:

$$\tilde{G}^{(0)}(\boldsymbol{p},p_z|\omega)=1/[\mathrm{i}\omega_\nu-\xi_p-t_\perp\cos(p_zd)] \qquad (9)$$

where $\omega_\nu=\pi T(2\nu+1)$ is a Matsubara frequency, T being temperature with $\nu=0,\pm1,\pm2,\ldots$; $\xi_p=v_{\mathrm{F}}(|p|-p_{\mathrm{F}})$, where v_{F} and p_{F} are Fermi velocity and Fermi momentum, respectively. We perform the Fourier transform of (9) as

$$\tilde{G}^{(0)}_{j-j'}(\boldsymbol{p}|\omega)=\int_{-\pi/d}^{\pi/d}\frac{\mathrm{d}p_z}{2\pi}\tilde{G}^{(0)}(\boldsymbol{p},p_z|\omega)\exp[\mathrm{i}p_z(j-j')d]. \qquad (10)$$

To study the weak superconductivity for Josephson-like interlayer coupling under the condition $t_\perp<kT_c^{(0)}<\mathcal{E}_{\mathrm{F}}$, we expand $\tilde{G}^{(0)}(\boldsymbol{p},p_z|\omega)$ in (10) over the small parameter $t_\perp/kT<1$ in the vicinity of the critical temperature T_c and obtain

$$\tilde{G}^{(0)}_{j-j'}(\boldsymbol{p}|\omega)=[(1/2)t_\perp g^{(0)}(\boldsymbol{p},\omega)]^{|j-j'|}g^{(0)}(\boldsymbol{p},\omega)+O(((1/2)t_\perp g^{(0)})^{|j-j'|+2}) \qquad (11)$$

where $g^{(0)}(\boldsymbol{p},\omega)$ is the 'bare' Green function, which corresponds to 2D motion of an electron and is given as

$$g^{(0)}(\boldsymbol{p},\omega)=1/(\mathrm{i}\omega_\nu-\xi_p). \qquad (12)$$

The formula (11) shows that the Green function $\tilde{G}^{(0)}_{j-j'}(\boldsymbol{p}|\omega)$ falls with increasing distance $z=d|j-j'|$ between layers j and j'.

By introducing the order parameters $\Delta_0^*(\boldsymbol{r};j,j)$ for the intralayer and $\Delta_{\pm1}^*(\boldsymbol{r};j,j\pm1)$ for the interlayer as

$$\Delta_0^*(\boldsymbol{r};j,j)=|V_0|T\sum_\omega F^+_{jj}(\boldsymbol{r},\boldsymbol{r}|\omega) \qquad (13a)$$

$$\Delta_{\pm1}^*(\boldsymbol{r};j,j\pm1)=|V_1|T\sum_\omega F^+_{j,j\pm1}(\boldsymbol{r},r|\omega) \qquad (13b)$$

we get the following integro-differential equations from (7) and (8):

$$G_{jj'}(\boldsymbol{r},\boldsymbol{r}'|\omega)=G^{(0)}_{jj'}(\boldsymbol{r},\boldsymbol{r}'|\omega)-\sum_{k=0,\pm1}\sum_i\int\mathrm{d}^2r_1\,G^{(0)}_{ji}(\boldsymbol{r},\boldsymbol{r}_1|\omega)\Delta_k(\boldsymbol{r}';i,i+k)F^+_{i+k,j'}(\boldsymbol{r},\boldsymbol{r}'|\omega) \qquad (14)$$

$$F^+_{jj'}(\boldsymbol{r},\boldsymbol{r}'|\omega)=\sum_{k=0,\pm1}\sum_i\int\mathrm{d}^2r_1\,G^{(0)}_{ij}(\boldsymbol{r}_1,\boldsymbol{r}|-\omega)\Delta^*_k(\boldsymbol{r}_1;i,i+k)G_{i+k,j'}(\boldsymbol{r},\boldsymbol{r}'|\omega). \qquad (15)$$

Here $G^{(0)}_{jj'}(\boldsymbol{r},\boldsymbol{r}'|\omega)$ is the 'bare' Green function in the presence of a magnetic field.

In the vicinity of the critical temperature we derive the equations for the order parameters up to third order in $\Delta^*_l(\boldsymbol{r};j,j+1)$ from the above equations:

$$\frac{\Delta^*_l(\boldsymbol{r};j,j+1)}{|V_l|} = T\sum_{\omega}\sum_{k=0,\pm1}\sum_{i}\int \mathrm{d}^2r_1\, G^{(0)}_{ij}(\boldsymbol{r}_1,\boldsymbol{r}|-\omega)\Delta^*_k(\boldsymbol{r}_1;i,i+k)G^{(0)}_{i+k,j+l}(\boldsymbol{r}_1,\boldsymbol{r}|\omega)$$
$$-T\sum_{\omega}\sum_{\substack{k,k_1\\k_2=0,\pm1}}\sum_{\substack{i,i_1\\i_2}}\int \mathrm{d}^2r_1\int \mathrm{d}^2r_2\int \mathrm{d}^2r_3\, G^{(0)}_{ij}(\boldsymbol{r}_1,\boldsymbol{r}|-\omega)\Delta^*_k(\boldsymbol{r}_1;i,i+k)$$
$$\times G^{(0)}_{i+k_1,i_1}(\boldsymbol{r}_1,\boldsymbol{r}_2)|\omega)\Delta_{k_1}(\boldsymbol{r}_2;i_1,i_1+k_1)G^{(0)}_{i_2,i_1+k_1}(\boldsymbol{r}_3,\boldsymbol{r}_2|-\omega)\Delta^*_{k_2}(\boldsymbol{r}_3;i_2,i_2+k_2)$$
$$\times G^{(0)}_{i_2+k_2,j+l}(\boldsymbol{r}_3,\boldsymbol{r}|\omega)+\ldots \tag{16}$$

where $l=0,\pm1$ and $V_{-1}\equiv V_1$.

A magnetic-field dependence of the Green function in (16) is provided in a quasiclassical approximation [27] as

$$G^{(0)}_{i,j}(\boldsymbol{r},\boldsymbol{r}'|\omega) = \bar{G}^{(0)}_{i-j}(\boldsymbol{r}-\boldsymbol{r}'|\omega)\exp\left[-\frac{\mathrm{i}e}{\hbar c}\left(\int_{id}^{jd}A_z\,\mathrm{d}\kappa+\int_{\boldsymbol{r}}^{\boldsymbol{r}'}\boldsymbol{A}(\boldsymbol{s})\,\mathrm{d}\boldsymbol{s}\right)\right]. \tag{17}$$

According to expression (11) for the 'bare' Green function $\bar{G}^{(0)}_{i-j}(\boldsymbol{r}-\boldsymbol{r}'|\omega)$, we shall keep all terms up to the second order of the interlayer tunnelling integral, $t_\perp$, in the sum over i in (16). Besides, in accordance with the order-parameter symmetry we believe that

$$\Delta^*_{-1}(\boldsymbol{r};j,j-1)=\Delta^*_1(\boldsymbol{r};j-1,j)\equiv\Delta^*_1(\boldsymbol{r},j-1). \tag{18}$$

After the replacement of $\Delta^*_1(\boldsymbol{r},j)$ by $(1/\sqrt{2})\Delta^*_1(\boldsymbol{r},j)$, the following equations for Δ^*_0 and Δ^*_1 are obtained:

$$\frac{\Delta^*_0(\boldsymbol{r},j)}{|V_0|} = (K^0_{00}+2K^0_{11})\Delta^*_0(\boldsymbol{r},j) - K^0_{11}\left[2\Delta^*_0(\boldsymbol{r},j)-\exp\left(\mathrm{i}\frac{2ed}{\hbar c}A_z\right)\Delta^*_0(\boldsymbol{r},j+1)\right.$$
$$\left.-\exp\left(-\mathrm{i}\frac{2ed}{\hbar c}A_z\right)\Delta^*_1(\boldsymbol{r},j-1)\right]+\sqrt{2}K^0_{01}\left[\exp\left(\mathrm{i}\frac{ed}{\hbar c}A_z\right)\Delta^*_1(\boldsymbol{r},j)\right.$$
$$\left.+\exp\left(-\mathrm{i}\frac{ed}{\hbar c}A_z\right)\Delta^*_1(\boldsymbol{r},j-1)\right]+(K^1_{00}+2K^1_{11})\left(\frac{\partial}{\partial\boldsymbol{r}}+\mathrm{i}\frac{2e}{\hbar c}\boldsymbol{A}\right)^2\Delta^*_0(\boldsymbol{r},j)$$
$$+\sqrt{2}K^1_{01}\left(\frac{\partial}{\partial\boldsymbol{r}}+\mathrm{i}\frac{2e}{\hbar c}\boldsymbol{A}\right)^2\left[\exp\left(\mathrm{i}\frac{ed}{\hbar c}A_z\right)+\exp\left(-\mathrm{i}\frac{ed}{\hbar c}A_z\right)\right]\Delta^*_1(\boldsymbol{r},j)+I^0_3+\ldots \tag{19}$$

$$\frac{\Delta^*_1(\boldsymbol{r},j)}{|V_1|} = (K^0_{00}+3K^0_{11}+2k^0_{02})\Delta^*_1(\boldsymbol{r},j)-(K^0_{11}+K^0_{02})\left[2\Delta^*_1(\boldsymbol{r},j)\right.$$
$$\left.-\exp\left(-\mathrm{i}\frac{2ed}{\hbar c}A_z\right)\Delta^*_1(\boldsymbol{r},j-1)-\exp\left(\mathrm{i}\frac{2ed}{\hbar c}A_z\right)\Delta^*_1(\boldsymbol{r},j+1)\right]$$
$$+\sqrt{2}K^0_{01}\left[\exp\left(-\mathrm{i}\frac{ed}{\hbar c}A_z\right)\Delta^*_0(\boldsymbol{r},j)+\exp\left(\mathrm{i}\frac{ed}{\hbar c}A_z\right)\Delta^*_0(\boldsymbol{r},j+1)\right]$$
$$+(K^1_{00}+3K^1_{11}+2K^1_{02})\left(\frac{\partial}{\partial\boldsymbol{r}}+\mathrm{i}\frac{2e}{\hbar c}\boldsymbol{A}\right)^2\Delta^*_1(\boldsymbol{r},j)+\sqrt{2}K^1_{01}\left(\frac{\partial}{\partial\boldsymbol{r}}+\mathrm{i}\frac{2e}{\hbar c}\boldsymbol{A}\right)^2$$
$$\times\left[\exp\left(+\mathrm{i}\frac{ed}{\hbar c}A_z\right)+\exp\left(-\mathrm{i}\frac{ed}{\hbar c}A_z\right)\right]\Delta^*_0(\boldsymbol{r},j)+I^1_3+\ldots. \tag{20}$$

The third-order terms in the order parameters in equations (19) and (20) are denoted by I_3^0 and I_3^1. The expressions of I_3^0 and I_3^1 are given in appendix 1. The coefficients K_{ij} in equations (19) and (20) are defined as follows:

$$K_{ij}^0 = T\sum_{\omega}\int d^2r_1\, \bar{G}_i^{(0)}(r_1 - r|-\omega)\bar{G}_j^{(0)}(r_1 - r|\omega)$$

$$K_{ij}^1 = \tfrac{1}{2}T\sum_{\omega}\int d^2r_1\, \bar{G}_i^{(0)}(r_1 - r|-\omega)\bar{G}_j^{(0)}(r_1 - r|\omega)(r_1 - r)^2$$

and

$$K_{00}^0 = \nu_0^{2d}\{\ln(\omega_D/T) + \tfrac{1}{2}(t_\perp/4kT)^2[1 + (2T/\omega_D)^2]\} \tag{21a}$$

$$K_{11}^0 = (\nu_0^{2d}/16)(t_\perp/2kT)^2[1 - (2T/\omega_D)^2] \tag{21b}$$

$$K_{02}^0 = -(\nu_0^{2d}/32)(t_\perp/2kT)^2[1 + (2T/\omega_D)^2] \tag{21c}$$

$$K_{00}^1 = \mathcal{E}_F/[32\pi(kT)^2] \tag{21d}$$

$$K_{11}^1 = -(t_\perp/4kT)^2\mathcal{E}_F/[64\pi(kT)^2] \tag{21e}$$

where $\nu_0^{2d} = m/(2\pi\hbar^2)$ is the density of the 2D electron states and ω_D is the Debye temperature. We should note that the coefficient K_{01}^0 depends on the filling degree of the 2D electron band; $K_{01}^0 = 0$ corresponds to the half-filled case. This fact was also shown by previous authors [21, 22]. K_{01}^0 can be represented in the following form:

$$K_{01}^0 = \frac{\nu_0^{2d}}{8}\frac{t_\perp}{\mathcal{E}_F}\left[\frac{1}{1-\mu_\parallel}\tanh\left(\frac{\mathcal{E}_F}{2kT}(1-\mu_\parallel)\right) - \tanh\left(\frac{\mathcal{E}_F}{2kT}\right)\right] \tag{22}$$

where the parameter $\mu_\parallel$ characterizes the degree of deviation from half-filling and $0 < \mu_\parallel < 1$. For metals usually $\mu_\parallel \ll 1$ and $\mu_\parallel = 0$ corresponds to a half-filled 2D electron band.

By using the gap equations (19) and (20) the Ginzburg–Landau functional with two order parameters can be constructed:

$$\begin{aligned}
F\{\Delta_0, \Delta_1\} = \sum_j \int d^2r \Bigg\{ & \alpha_0(T)|\Delta_0(r, j)|^2 + \alpha_1(T)|\Delta_1(r, j)|^2 + E_\perp \sum_{g=\pm 1}\Bigg|\Delta_0(r, j) \\
& - \exp\left(-i\frac{2ed}{\hbar c}gA_z\right)\Delta_0(r, j+g)\Bigg|^2 + \frac{\hbar^2}{4m}\left|\left(\frac{\partial}{\partial r} - i\frac{2e}{\hbar c}A(r, j)\right)\Delta_0(r, j)\right|^2 \\
& + \tfrac{1}{2}E_\perp\sum_{g=\pm 1}\left|\Delta_1(r, j) - \exp\left(-i\frac{2ed}{\hbar c}gA_z\right)\Delta_1(r, j+g)\right|^2 \\
& + \frac{\hbar^2}{4m}\left|\left(\frac{\partial}{\partial r} - i\frac{2e}{\hbar c}A(r, j)\right)\Delta_1(r, j)\right|^2 - E_{01}\Bigg\{\Delta_1(r, j)\exp\left(-i\frac{ed}{\hbar c}A_z\right) \\
& \times\left[\exp\left(i\frac{2ed}{\hbar c}A_z\right)\Delta_0^*(r, j+1) - \Delta_0^*(r, j)\right] - \Delta_0(r, j) \\
& \times\left[\exp\left(i\frac{ed}{\hbar c}A_z\right)\Delta_1^*(r, j) - \exp\left(-i\frac{ed}{\hbar c}A_z\right)\Delta_1^*(r, j-1)\right] + \mathrm{HC}\Bigg\} \\
& - 4E_{01}\Bigg[\exp\left(-i\frac{ed}{\hbar c}A_z\right)\Delta_1(r, j)\Delta_0^*(r, j) \\
& + \exp\left(i\frac{ed}{\hbar c}A_z\right)\Delta_1^*(r, j)\Delta_0(r, j)\Bigg]\Bigg\} + \Delta F_4 + \int d^2r\int dz\,\frac{H^2}{8\pi}
\end{aligned} \tag{23}$$

where ΔF_4 denotes the fourth-order terms in the order parameters in equation (23). The expression of ΔF_4 is given in appendix 1. The coefficients $\alpha_0(T)$ and $\alpha_1(T)$ in (23) are defined as

$$\alpha_0(T) = 4\frac{(kT)^2}{\mathcal{E}_F}\left[\ln\left(\frac{T}{T_{c0}}\right) + \left(\frac{t_\perp}{4kT}\right)^2 \frac{T-T_{c0}}{T}\right] \simeq 4\frac{(kT)^2}{\mathcal{E}_F}\left[1 + \left(\frac{t_\perp}{4kT_{c0}}\right)^2\right]\frac{T-T_{c0}}{T_{c0}} \tag{24}$$

and

$$\alpha_1(T) = 4\frac{(kT)^2}{\mathcal{E}_F}\left[\ln\left(\frac{T}{T_{c1}}\right) + \left(\frac{t_\perp}{4kT}\right)^2 \frac{T-T_{c1}}{T_{c1}}\right] \simeq 4\frac{(kT)^2}{\mathcal{E}_F}\left[1 + \left(\frac{t_\perp}{4kT_{c1}}\right)^2\right]\frac{T-T_{c1}}{T_{c1}}. \tag{25}$$

Here T_{c0} and T_{c1} are the critical temperatures, corresponding to intra- and interlayer pairings. These critical temperatures are given by the following expressions:

$$T_{c0} = \omega_D \exp(-1/\nu_0^{2d}|V_0) \qquad \text{and} \qquad T_{c1} = \omega_D \exp(-1/\nu_0^{2d}|V_1|). \tag{26}$$

The third and fifth terms in $F\{\Delta_0, \Delta_1\}$ characterize the tunnelling of intra- and interlayer pairs between nearest-neighbouring planes and tunnelling strength

$$E_\perp = t_\perp^2/32\mathcal{E}_F. \tag{27}$$

It must be stressed that the tunnelling energies of intra- and interlayer pairs are different (see equation (23)). Introducing the transverse components of the mass tensor for intra- and interlayer pairs as $M_0 = \hbar^2/2E_\perp d^2$ and $M_1 = \hbar^2/E_\perp d^2$, respectively, they are seen to differ by a factor of $1/2$.

By setting $\Delta_1(\boldsymbol{r}, j)$ to zero in equation (23), we get the Lawrence–Doniach free-energy functional [8] for the Josephson-coupled quasi-2D SC. The coefficient E_{01} is represented as

$$E_{01} = \frac{1}{2\sqrt{2}}\left(\frac{kT}{\mathcal{E}_F}\right)^2 t_\perp \left[\frac{1}{1-\mu_\parallel}\tanh\left(\frac{\mathcal{E}_F}{2kT}(1-\mu_\parallel)\right) - \tanh\left(\frac{\mathcal{E}_F}{2kT}\right)\right]. \tag{28}$$

In the continuum limit, where j is replaced by z, the linear derivative of the order parameters with respect to z can be obtained as

$$(\Delta_1 \partial\Delta_0^*/\partial z - \Delta_0 \partial\Delta_1^*/\partial z + \text{HC})$$

instead of the seventh term in $F\{\Delta_0, \Delta_1\}$. As is known, the linear derivative of the order parameter in an expression for the free-energy functional is called the Lifshitz invariant.

3. The upper critical magnetic field

Following Gor'kov's equations (19) and (20) it is possible to find the upper critical magnetic field, H_{c2}, in the vicinity of the transition temperature. We shall study both components of H_{c2}, parallel and perpendicular to the layers, separately.

The choice of the vector potential A as $A = \{0, xH, 0\}$ corresponds to the perpendicular component of the magnetic field H. Excluding the coordinate y and the discrete layer number j, the equations (19) and (20) can be reduced to

$$\{-(\hbar^2/4m)[\partial^2/\partial x^2 - (x/l_s)^2] + \alpha_0(T)\}\Delta_0^*(x) - 4E_{01}\Delta_1^*(x) = 0 \tag{29}$$

and

$$\{-(\hbar^2/4m)[\partial^2/\partial x^2 - (x/l_s)^2] + \alpha_1(T)\}\Delta_1^*(x) - 4E_{01}\Delta_0^*(x) = 0 \tag{30}$$

where $l_s^2 = \hbar c/2eH$ is the cooperon magnetic length.

By elimination we can get equations for $\Delta_0^*(x)$ and $\Delta_1^*(x)$ from (29) and (30), which turn out to be identical. Therefore we can suppose that

$$\Delta_0^*(x) = C\Delta_1^*(x)$$

where C is a constant.

The coefficient C is obtained by joint solution of the oscillator equations (29) and (30) as

$$l_s^2(4m/\hbar^2)[-\alpha_0(T) + C4E_{01}] = 2n + 1$$

$$l_s^2(4m/\hbar^2)[-\alpha_1(T) + (1/C)4E_{01}] = 2n + 1.$$

$H_{c2}^{\perp}$ satisfies this system of equations for $n = 0$. As a result

$$H_{c2}^{\perp} = (cm/e\hbar)\{-[\alpha_0(T) + \alpha_1(T)] + [(\alpha_0 - \alpha_1)^2 + (8E_{01})^2]^{1/2}\}. \tag{31}$$

The right-hand side of equation (31) vanishes at temperatures $T_c^* > T_{c0}, T_{c1}$. This is a 'true' critical temperature estimated by mean-field approximation for the system under consideration. As shown by many authors [18–22], the existence of two order parameters in the system leads to an increase of the critical temperature.

For parallel magnetic field, we choose $H = \{0, H, 0\}$ and $A = \{0, 0, -xH\}$. Then, Gor'kov's equations (19) and (20) are reduced to the following form:

$$[-(\hbar^2/4m)\partial^2/\partial x^2 - 4E_\perp \cos(xd/l_s^2) + \alpha_0 + 4E_\perp]\Delta_0^*(x) - 4E_{01}\cos(xd/2l_s^2)\Delta_1^*(x) = 0 \tag{32}$$

$$[-(\hbar^2/4m)\partial^2/\partial x^2 - 2E_\perp \cos(xd/l_s^2) + \alpha_1 + 2E_\perp]\Delta_1^*(x) - 4E_{01}\cos(xd/2l_s^2)\Delta_0^*(x) = 0. \tag{33}$$

The equations (32) and (33) are similar to Hill's equation (28). They are solved for some asymptotic cases.

In a weak magnetic field satisfying the condition $H^2 < (16mc^2/e^2d^2)\max\{E_\perp, E_{01}\}$, the following expression for $H_{c2}^{\parallel}$ is obtained (see appendix 2):

$$H_{c2}^{\parallel} = \frac{cm}{2e\hbar}\frac{1}{2ab(a+b)}\left(\frac{\hbar^2}{2md^2E_\perp}\right)^{1/2} [\![-[(1+2ab)\alpha_0(T) + (2+2ab)\alpha_1(T) + 4E_{01}^2/E_\perp] + \{[(1+2ab)\alpha_0 + (2+2ab)\alpha_1 + 4E_{01}^2/E_\perp]^2 - 16ab(a+b)^2(\alpha_0\alpha_1 - 16E_{01}^2)\}^{1/2}]\!] \tag{34}$$

where a and b are positive constants,

$$a = \tfrac{1}{2}\{3 + [1 + (E_{01}/E_\perp)^2]^{1/2}\}^{1/2}$$

$$b = \tfrac{1}{2}\{3 - [1 + (E_{01}/E_\perp)^2]^{1/2}\}^{1/2}.$$

As shown in appendix 2, the ratio $E_{01}/E_\perp$ varies in such a range where b always takes positive values. E_{01} characterizes the coupling energy (see (A2.8) and (A2.9)) between the two oscillators. The values of E_{01} should not be too large to destroy the cooperons' oscillations.

The system of equations (32) and (33) may be solved approximately for a magnetic field satisfying the condition $(16mc^2/e^2d^2)E_{01} < H^2 < (16mc^2/e^2d^2)E_\perp$ with the assumption that $E_{01} < E_\perp$ (see appendix 2). In this interval we obtain

$$H_{c2}^{\parallel} = (c^2m/2e^2d^2E_\perp)^{1/2}[-\alpha_0(T) + 4E_{01}] \tag{35}$$

$$H_{c2}^{\parallel} = (c^2m/e^2d^2E_\perp)^{1/2}[-\alpha_1(T) + 4E_{01}]. \tag{36}$$

For $T_{c0} > T_{c1}$ the experimentally observed H_{c2} will be defined by (35).

For higher magnetic field values such as $H^2 > (16mc^2/e^2d^2)\max\{E_\perp, E_{01}\}$ the linear dependence of $H_{c2}^{\parallel}$ on T is no longer possible. For these values of H, the temperature dependence of $H_{c2}^{\parallel}$ takes the form

$$(H_{c2}^{\parallel})^2 = \frac{8mc^2}{e^2d^2}\frac{E_\perp^2 + 4E_{01}^2}{4E_\perp + \alpha_0(T)} \tag{37}$$

and

$$(H_{c2}^{\parallel})^2 = \frac{4mc^2}{e^2d^2}\frac{E_\perp^2 + 8E_{01}^2}{2E_\perp + \alpha_0(T)}. \tag{38}$$

The larger of these two expressions would be the actually observed $H_{c2}^{\parallel}$.

The expression (37) (or (38)) indicates that $H_{c2}^{\parallel}$ becomes infinite at the temperature $T = T^*$ satisfying the equation $-\alpha_0(T^*) = 4E_\perp$ (or $-\alpha_1(T^*) = 2E_\perp$). That is, the orbital depairing effect of a magnetic field parallel to the layers does not destroy the superconductivity. In this case the cores of the vortices fit between the SC layers and the external magnetic field has no effects on the superconductivity. The divergence of $H_{c2}^{\parallel}$ at T^* is removed when the paramagnetic effect of Chandrasekhar and Clogston [29, 30] or spin–orbit scattering [10] is taken into account. It is worth noting that this effect also takes place in the theory of a layered SC with Josephson-like interlayer coupling, provided that electron pairing occurs only inside each layer [7, 9, 10].

4. Conclusion

The Ginzberg–Landau free-energy functional for layered superconductors with two order parameters was obtained for $t_\perp < kT_c^{(0)}$ by applying the Gor'kov–Nambu theory. This functional contains several new terms such as Lifshitz's invariant and terms corresponding to the Josephson tunnelling of both intra- and interlayer pairs, in addition to the terms present in an isotropic SC functional. The existence of these terms in the free-energy functional drastically changes the magnetic properties of the system. For the parallel component of the magnetic field at $T = T^* < T_c$, the normal cores of the vortices fit between the layers, allowing them to remain superconducting. This effect takes place also in a Josephson-coupled layered SC with only intralayer pairing [9, 10]. Such behaviour of H_{c2}^* makes the understanding of positive curvature in the temperature dependence of H_{c2} possible [4, 31].

The free-energy functional (23) obtained for layered superconductors with two order parameters allows the study of the spectrum of collective excitations and the effects of the order-parameter phase fluctuations.

Using the Hamiltonian formalism with Coulomb effects, the system of non-linear coupled equations for phases $\phi_0(\boldsymbol{r}, j; t)$ and $\phi_1(\boldsymbol{r}, j; t)$ may be obtained as

$$\begin{aligned}\frac{\mathrm{d}^2\phi_0(\boldsymbol{r}, j; t)}{\mathrm{d}t^2} &= \frac{N_{s0}(T)}{mC_0\xi_\parallel^2}\left(\frac{\partial^2\phi_0(\boldsymbol{r}, j; t)}{\partial x^2} + \frac{\partial^2\phi_0(\boldsymbol{r}, j; t)}{\partial y^2}\right)\\ &\quad - 2\frac{N_{s0}(T)}{\hbar^2 C_0}E_\perp\{\sin[\phi_0(\boldsymbol{r}, j) - \phi_0(\boldsymbol{r}, j+1)] + \sin[\phi_0(\boldsymbol{r}, j) - \phi_0(\boldsymbol{r}, j-1)]\}\\ &\quad - 4\frac{(N_{s0}N_{s1})}{\hbar^2 C_0}E_{01}\{\sin[\phi_0(\boldsymbol{r}, j) - \phi_1(\boldsymbol{r}, j-1)] + \sin[\phi_0(\boldsymbol{r}, j) - \phi_1(\boldsymbol{r}, j)]\}\end{aligned} \tag{39}$$

$$\begin{aligned}\frac{\mathrm{d}^2\phi_1(\boldsymbol{r}, j; t)}{\mathrm{d}t^2} &= \frac{N_{s1}(T)}{mC_1\xi_\parallel^2}\left(\frac{\partial^2\phi_1(\boldsymbol{r}, j; t)}{\partial x^2} + \frac{\partial^2\phi_1(\boldsymbol{r}, j; t)}{\partial y^2}\right)\\ &\quad - \frac{N_{s1}(T)}{\hbar^2 C_1}E_\perp\{\sin[\phi_1(\boldsymbol{r}, j) - \phi_1(\boldsymbol{r}, j+1)] + \sin[\phi_1(\boldsymbol{r}, j) - \phi_1(\boldsymbol{r}, j-1)]\}\\ &\quad - 4\frac{(N_{s0}N_{s1})}{\hbar^2 C_1}E_{01}\{\sin[\phi_1(\boldsymbol{r}, j) - \phi_0(\boldsymbol{r}, j+1)] + \sin[\phi_1(\boldsymbol{r}, j) - \phi_0(\boldsymbol{r}, j)]\}\end{aligned} \tag{40}$$

where $\xi_\parallel$ is the coherence length inside the SC layers, $N_{s0}(T) = 2|\Delta_0|^2$ and C_0 ($N_{s1}(T) = 2|\Delta_1|^2$ and C_1) are the density of superconducting electrons and the capacitance matrix for an intralayer (interlayer) pair.

For half-filling inside the SC layers, the equations (39) and (40) are reduced to $2N$ sine–Gordon-like coupled equations, N being the number of layers. The static distribution of the phases ϕ_0 and ϕ_1 for $t_\perp \to 0$ (in this case both the coefficients $E_\perp$ and E_{01} vanish) will be described by two Laplace's equations for each layer, j. In other words, the SC layers are uncoupled in $t_\perp \to 0$, and a superconducting state should be described by the two kinds of Kosterlitz–Thouless topological defects [16, 17]. Indeed, for $t_\perp \to 0$, the mixing of the order parameters does not occur in the functional (23) and the free-energy functional becomes effectively two-dimensional. Therefore, the order-parameter phase fluctuations should strongly affect the superconducting properties of the system.

In addition to the interlayer electron–electron attractive interaction ($2c$) given in the particle–hole channel in the Hamiltonian (2), there is another term corresponding to an interlayer interaction in the particle–particle channel. The influence of both these terms on ODLRO is under investigation. The behaviour of the correlation function and some features of ODLRO for a layered SC with two order parameters will be the subject of a future work.

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Appendix 1

The common expression of I_3^l ($l = 0, 1$) is given by the last term in (16). Summing over k, k_1 and k_2 in (16) and using the symmetry condition of (18), we retain such terms in sums over i, i_1 and i_2 in which all order parameters depend only on the jth layer number. After the replacement of $\Delta_1^*(\boldsymbol{r}, j)$ by $(1/\sqrt{2})\Delta_1^*(\boldsymbol{r}, j)$ in (16) the following expressions for I_3^0 and I_3^1 are obtained:

$$I_3^0 = A_0(|\Delta_0|^2+|\Delta_1|^2)\Delta_0^*+2\sqrt{2}A_1|\Delta_0|^2\Delta_1^*+\sqrt{2}A_1|\Delta_1|^2\Delta_1^*+\sqrt{2}A_1(\Delta_0^*)^2\Delta_1+A_2(\Delta_1^*)^2\Delta_0 \tag{A1.1}$$

$$I_3^1 = A_0(\tfrac{1}{2}|\Delta_1^2|+|\Delta_0^2|)\Delta_1^*+2\sqrt{2}A_1|\Delta_1|^2\Delta_0^*+\sqrt{2}A_1|\Delta_1^*|^2\Delta_0+\sqrt{2}A_1(\Delta_1^*)^2\Delta_0+A_2(\Delta_0^*)^2\Delta_1 \tag{A1.2}$$

where

$$A_0 = -T\sum_{\omega}\int\int\int \mathrm{d}^2r_1\,\mathrm{d}^2r_2\,\mathrm{d}^2r_3\,G_{jj}^0(\boldsymbol{r}_1,\boldsymbol{r}|-\omega)G_{jj}^0(\boldsymbol{r}_1,\boldsymbol{r}_2|\omega)G_{jj}^0(\boldsymbol{r}_3,\boldsymbol{r}_2|-\omega)$$
$$\times G_{jj}^0(\boldsymbol{r}_3,\boldsymbol{r}|\omega) = -\frac{\nu_0^{2\mathrm{d}}}{16(kT)^2} \tag{A1.3}$$

$$A_1 = -T\sum_{\omega}\int\int\int \mathrm{d}^2r_1\,\mathrm{d}^2r_2\,\mathrm{d}^2r_3\,G_{jj}^0(\boldsymbol{r}_1,\boldsymbol{r}|-\omega)G_{jj}^0(\boldsymbol{r}_1,\boldsymbol{r}_2|\omega)G_{jj}^0(\boldsymbol{r}_3,\boldsymbol{r}_2|-\omega)$$
$$\times G_{j+1,j}^0(\boldsymbol{r}_3,\boldsymbol{r}|\omega) = -\frac{\nu_0^{2\mathrm{d}}}{64}\frac{t_\perp}{kT\mathcal{E}_\mathrm{F}^2}\left[\frac{1}{(1-\mu_\parallel)^2}\cosh^{-2}\left(\frac{\mathcal{E}_\mathrm{F}}{2kT}(1-\mu_\parallel)\right)\right.$$
$$-\cosh^{-2}\left(\frac{\mathcal{E}_\mathrm{F}}{2kT}\right)-\frac{2kT}{\mathcal{E}_\mathrm{F}(1-\mu_\parallel)^3}\tanh\left(\frac{\mathcal{E}_\mathrm{F}}{2kT}(1-\mu_\parallel)\right)$$
$$\left.+\frac{2kT}{\mathcal{E}_\mathrm{F}}\tanh\left(\frac{\mathcal{E}_\mathrm{F}}{2kT}\right)\right] \tag{A1.4}$$

$$A_2 = -T\sum_{\omega}\int\int\int \mathrm{d}^2r_1\,\mathrm{d}^2r_2\,\mathrm{d}^2r_3\,G_{jj}^0(\boldsymbol{r}_1,\boldsymbol{r}|-\omega)G_{jj}^0(\boldsymbol{r}_1,\boldsymbol{r}_2|\omega)G_{j,j+1}^0(\boldsymbol{r}_3,\boldsymbol{r}_2|-\omega)$$
$$\times G_{j+1,j}^0(\boldsymbol{r}_3,\boldsymbol{r}|\omega) = -\frac{\nu_0^{2\mathrm{d}}t_\perp^2}{(4kT)^4} \tag{A1.5}$$

The functional integration of (19) and (20) gives the Ginzburg–Landau free-energy functional of (23), where ΔF_4 is expressed as

$$\Delta F_4 = \sum_j\int \mathrm{d}^2r\,\{\tfrac{1}{2}\beta_0|\Delta_0(\boldsymbol{r},j)|^4+\tfrac{1}{4}\beta_0|\Delta_1(\boldsymbol{r},j)|^4+\beta_0|\Delta_0|^2|\Delta_1|^2+\beta_1(|\Delta_0|^2+|\Delta_1|^2)$$
$$\times(\Delta_0^*\Delta_1+\Delta_1^*\Delta_0)+\tfrac{1}{2}\beta_2[(\Delta_0^*)^2\Delta_1^2+(\Delta_1^*)^2\Delta_0^2]\} \tag{A1.6}$$

where

$$\beta_0 = (1/\nu^{2\mathrm{d}})(kT/\mathcal{E}_\mathrm{F})^2 \tag{A1.7}$$

$$\beta_1 = \frac{1}{2\sqrt{2}\nu_0^{2\mathrm{d}}}\frac{t_\perp(kT)^3}{\mathcal{E}_\mathrm{F}^4}\left[\frac{1}{(1-\mu_\parallel)^2}\cosh^{-2}\left(\frac{\mathcal{E}_\mathrm{F}}{2kT}(1-\mu_\parallel)\right)-\cosh^{-2}\left(\frac{\mathcal{E}_\mathrm{F}}{2kT}\right)\right.$$
$$\left.-\frac{2kT}{\mathcal{E}_\mathrm{F}(1-\mu_\parallel)^3}\tanh\left(\frac{\mathcal{E}_\mathrm{F}}{2kT}(1-\mu_\parallel)\right)+\frac{2kT}{\mathcal{E}_\mathrm{F}}\tanh\left(\frac{\mathcal{E}_\mathrm{F}}{2kT}\right)\right] \tag{A1.8}$$

$$\beta_2 = (1/16\nu^{2\mathrm{d}})(t_\perp/\mathcal{E}_\mathrm{F})^2. \tag{A1.9}$$

Appendix 2

The system of equations (32) and (33) are analogous to Hill's equation. For $E_{0l} = 0$ they take the form of two uncoupled Mathieu equations [28]. This system of equations may be solved for some asymptotic cases. After the following replacements

$$(d/2l_s^2)x = t \qquad \text{and} \qquad \Delta_i^*(2l_s^2 t/d) = f_i(t) \qquad (\mathrm{i} = 0, 1) \qquad (\mathrm{A2.1})$$

equations (32) and (33) can be written in the following form:

$$[\lambda_0 + \eta\cos(2t) + \partial^2/\partial t^2]f_0(t) + \theta\cos(t)f_1(t) = 0 \qquad (\mathrm{A2.2})$$

$$[\lambda_1 + \tfrac{1}{2}\eta\cos(2t) + \partial^2/\partial t^2]f_1(t) + \theta\cos(t)f_0(t) = 0 \qquad (\mathrm{A2.3})$$

where

$$\lambda_0 = -(16ml_s^4/\hbar^2 d^2)[\alpha_0(T) + 4E_\perp] \qquad (\mathrm{A2.4})$$

$$\lambda_1 = -(16ml_s^4/\hbar^2 d^2)[\alpha_1(T) + 2E_\perp] \qquad (\mathrm{A2.5})$$

$$\eta = 64ml_s^4 E_\perp/\hbar^2 d^2 \qquad (\mathrm{A2.6})$$

$$\theta = 64ml_s^4 E_{01}/\hbar^2 d^2. \qquad (\mathrm{A2.7})$$

The case $\eta > 1$ and $\theta > 1$ corresponds to a weak magnetic field, i.e. $(2eH/\hbar c)^2 < (64m/d^2\hbar^2)\min\{E_\perp, E_{01}\}$.

Let the variable t of equations (A2.2) and (A2.3) be replaced by z as $t = z/(2\eta)^{1/4}$, then $f_i[z/(2\eta)^{1/4}] \equiv \tilde{f}_i(z)$ $(i = 0, 1)$. After expansion of cosines in (A1.2) and (A1.3) we keep all terms up to second power of z. Then we obtain two coupled oscillator equations:

$$\left(\frac{\partial^2}{\partial z^2} - z^2 + \frac{\lambda_0 + \eta}{(2\eta)^{1/2}}\right)\tilde{f}_0(z) + \frac{\theta}{2\eta}\left((2\eta)^{1/2} - \frac{z^2}{2}\right)\tilde{f}_1(z) = 0 \qquad (\mathrm{A2.8})$$

$$\left(\frac{\partial^2}{\partial z^2} - \frac{z^2}{2} + \frac{\lambda_1 + \eta/2}{(2\eta)^{1/2}}\right)\tilde{f}_1(z) + \frac{\theta}{2\eta}\left((2\eta)^{1/2} - \frac{z^2}{2}\right)\tilde{f}_0(z) = 0. \qquad (\mathrm{A2.9})$$

The solutions of equations (A2.8) and (A2.9) are assumed to be chosen as a linear combination of oscillator wavefunctions with lowest energies:

$$\tilde{f}_0(z) = A\mathrm{e}^{-az^2/2} + B\mathrm{e}^{-bz^2/2} \qquad (\mathrm{A2.10})$$

$$\tilde{f}_1(z) = C\mathrm{e}^{-az^2/2} + D\mathrm{e}^{-bz^2/2}. \qquad (\mathrm{A2.11})$$

Substitutions of (A2.10) and (A2.11) into (A2.8) and (A2.9) result in the following expressions for the coefficients a and b:

$$a = \tfrac{1}{2}\{3 + [1 + (\theta/\eta)^2]^{1/2}\}^{1/2} \qquad (\mathrm{A2.12})$$

$$b = \tfrac{1}{2}\{3 - [1 + (\theta/\eta)^2]^{1/2}\}^{1/2}. \qquad (\mathrm{A2.13})$$

Also the following relationships between the coefficients A, B and C, D are obtained:

$$C = (\eta/\theta)\{[1 + (\theta/\eta)^2]^{1/2} - 1\}A \tag{A2.14}$$

$$D = -(\eta/\theta)\{[1 + (\theta/\eta)^2]^{1/2} + 1\}B \tag{A2.15}$$

Since $\theta/\eta = E_{01}/E_{\perp}$, E_{01} and $E_{\perp}$ are expected to vary in such an interval, ensuring always $b \geqslant 0$. This means that the coupling energy ($\sim E_{01}$) of two oscillators in (A2.8) and (A2.9) cannot be too large to destroy the oscillation of 'cooperons'. Then, $H_{c2}^{\parallel}$ can be found as a solution of the determinant constructed by the coefficients of the system of equations obtained by the substitution of (A2.10) and (A2,11) in (A2.8) and (A2.9). As a result the expression (34) for $H_{c2}^{\parallel}$ is obtained.

For $\eta > 1$ and $\theta < 1$, the magnetic field is in an interval, given as

$$(16m/\hbar^2 d^2)E_{01} < (eH/\hbar c)^2 < (16m/\hbar^2 d^2)E_{\perp}.$$

The last terms in equations (A2.2) and (A2.3), which are proportional to θ, are taken as a small parameter. Then the expansions of $f_i(t)$ and λ_i ($i = 0, 1$) in θ are

$$f_i(t) = f_i^{(0)}(t) + \theta f_i^{(1)}(t) + \theta^2 f_i^{(2)}(t) + \cdots \tag{A2.16}$$

$$\lambda_i(t) = \lambda_i^{(0)}(t) + \theta\lambda_i^{(1)}(t) + \theta^2\lambda_i^{(2)}(t) + \cdots. \tag{A2.17}$$

Here,

$$f_0^{(0)}(t) = \exp[-\tfrac{1}{2}(2\eta)^{1/2}t^2] \qquad \text{and} \qquad f_1^{(0)}(t) = \exp(-\tfrac{1}{2}\eta^{1/2}t^2)$$

are the eigenfunctions of a linear oscillators in the lowest energetic state with the eigenvalues $\lambda_0^{(0)}$ and $\lambda_1^{(0)}$. After some routine calculations we get the following expressions for λ_0 and λ_1:

$$\lambda_0 = (2\eta)^{1/2} - \eta - \theta \tag{A2.18}$$

$$\lambda_1 = \eta^{1/2} - \tfrac{1}{2}\eta - \theta. \tag{A2.19}$$

From (A2.18) and (A2.19) we may derive the resulting relation for $H_{c2}^{\parallel}$:

$$(H_{c2}^{\parallel})^{(1)} = (cm/e\hbar)(\hbar^2/2md^2E_{\perp})^{1/2}[-\alpha_0(T) + 4E_{01}] \tag{A2.20}$$

$$(H_{c2}^{\parallel})^{(2)} = (cm/e\hbar)(\hbar^2/md^2E_{\perp})^{1/2}[-\alpha_1(T) + 4E_{01}]. \tag{A2.21}$$

For $T_{c1} > T_{c2}$ the first solution, (A2.20), must be chosen. For a strong magnetic field, i.e.

$$(2eH/\hbar c)^2 > (64m/d^2\hbar^2)\max\{E_{\perp}, E_{01}\}$$

the parameters θ and η in equations (A2.2) and (A2.3) turn out to be less than unity, i.e. $\theta < 1$ and $\eta < 1$. In this case $f_i(t)$ and λ_i in (A2.2) and (13) are expanded in both η and θ:

$$f_i(t) = f_i^{(00)}(t) + \eta f_i^{(01)}(t) + \theta f_i^{(10)}(t) + \eta\theta f_i^{(11)}(t) + \cdots \tag{A2.22}$$

$$\lambda_i(t) = n_i^2 + \eta\lambda_i^{01}(t) + \theta\lambda_i^{(10)}(t) + \eta\theta\lambda_i^{(11)}(t) + \cdots. \tag{A2.23}$$

Substituting (A2.22) and (A2.23) into equations (A2.2) and (A2.3) we may obtain each term of these expansions. Since the upper critical magnetic field corresponds to the lowest quantum numbers, we choose $n_0 = n_1 = 0$ and get the following results:

$$\lambda_0 = -\tfrac{1}{8}\eta^2 - \tfrac{1}{2}\theta^2 \tag{A2.24}$$

$$\lambda_1 = -\tfrac{1}{16}\eta^2 - \tfrac{1}{2}\theta^2. \tag{A2.25}$$

Taking into account the equations (A2.4)–(A2.7) the expressions for $H_{c2}^{\parallel}$ are obtained from (A2.24) and (A2.25):

$$(H_{c2}^{\parallel})^2 = \frac{8mc^2}{d^2e^2}\frac{E_\perp^2 + 4E_{01}^2}{\alpha_0(T) + 4E_\perp} \tag{A2.26}$$

and

$$(H_{c2}^{\parallel})^2 = \frac{4mc^2}{d^2e^2}\frac{E_\perp^2 + 8E_{01}^2}{\alpha_1(T) + 2E_\perp}. \tag{A2.27}$$

For $T_{c1} > T_{c2}$, $H_{c2}^{\parallel}$ will be defined by the equation (A2.26).

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